

Lucien Polak: friend and scholar

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I first met Lucien in 1963. I had joined the faculty at Imperial College in 1959, had cut my teeth on the exciting advances that marked the late 1950's and early 1960's (dynamic programming, the maximum principle, Kalman filtering) and was developing some of my own research interests in optimization and optimal control. John Westcott, then head of the control group, organised a visit to the US, funded by IBM, and I persuaded him to let me join him. Our first port of call was the University of California, Berkeley, which Lucien had joined after he had finished his Ph.D there. John and I were invited to join in a meeting on optimal control where the guests of honour were a few scientists from the Soviet Union and Lucien acted as interpreter. I had little inkling then that he would contribute so extensively to optimization and optimal control, let alone play so important a role in my life.

While our paths crossed from time to time (for example, we independently proposed, almost simultaneously, a method for estimation now called instrumental variables), our first sustained collaboration took place in 1972 at Imperial College. I had developed Differential Dynamic Programming, a computational approach to dynamic optimization, that was taken considerably further by a brilliant Ph.D student, David Jacobson. David's key insight was that, as in the proof of the maximum principle, strong variations could be profitably employed, in this case to determine a successor control. But we had not the muscle to develop a convergence theory for the rather unusual algorithms that performed excellently on constrained optimal control problems but had no conventional analogues in finite dimensional optimization. Lucien's excellent book *Computational Methods in Optimization* [1] appeared in

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1971 and his lucid exposition of algorithm models that ensured convergence convinced me he was the right person to collaborate with. The UK Science Research Council provided a Research Fellowship and we got to work. The successor control was constructed by switching between the current control and a new control obtained by minimization of the Hamiltonian; ‘step length’ was measured by the L_1 distance between the new and old controls, or by the measure of the time interval on which the two controls differed. Proving convergence was clearly an unusual exercise, but the power of Lucien’s algorithm models enabled us to consider non-classical problems and showed us how to modify the original step size strategy to establish a result that has remained unique to this day: any accumulation point generated by the algorithm satisfied the minimum principle (rather than a weak variation analogue of the necessary conditions of optimality for finite dimensional constrained optimization) [2]. Our collaboration had started successfully.

Our next collaboration was in Berkeley, in 1974 and established a common pattern of our future collaboration: visits (ultimately totalling 12 or more) to alternate between London and Berkeley, and to involve an extra-curricular activity, backpacking. Lucien, while in London, ordered Jo (my wife) and I to get fit, which we did by jogging, and subsequently guided us on a magnificent 3 day trip to Cathedral Peak in the Sierra’s, where we climbed to nearly 12,000 feet, slept in the open under unimaginably bright stars, guarded by a log fire (permitted then) and visited by a mother bear and her cub. Our scientific collaboration was equally successful. One project yielded a novel algorithm for the problem $\min\{f(x) \mid h(x) = 0, g(x) \geq 0\}$ by solving the inequality constrained problem $\min\{f(x) + ch(x) \mid h(x) \geq 0, g(x) \geq 0\}$ which we showed to be equivalent to the original problem if c was large enough (but finite). In effect, this is an early exact penalty function algorithm (a preliminary version had been presented in Naples in 1974) with objective function $f(x) + ch(x)$ replacing the objective function $f(x)$ and constraint $h(x) \leq 0$. This approach has been adopted by Andre Tits in his successful FSQP and interior point algorithms. A second paper [3] addressed a control design optimization problem with (infinite dimensional) constraints on frequency responses, an early contribution to semi-infinite programming.

Our collaboration continued in this vein, with hard research sessions every year or two in the UK or US, pleasantly interrupted by country hikes often terminated by a rendezvous at a pleasant restaurant, for 15 years until 1989. We repeatedly climbed the Sierras and hiked in many parts of England and Wales, from the Pennines and Yorkshire Moors, to the Black

Mountains and Devon coast. During this period we developed, inter alia, algorithms for optimal control [4, 5], an outer approximation algorithm for semi-infinite optimization problems arising in computer-aided design [6], a cut-map algorithm for the tolerancing problem in circuit design, and the first SQP algorithm [7] that successfully addressed the Maratos effect (by using a search arc) and included automatic adjustment of the penalty parameter, as well as research on non-differentiable optimization [8, 9], the solution of singular value inequalities [10] that arise in robust control design, and the use of barrier function methods for optimal control [11] and minimax problems [12]. To all our research Lucien brought an enquiring mind and a passion for rigour that successfully channelled my more wayward inspirations.

This collaboration, which was immensely rewarding to me and during which we wrote over 50 papers and delivered one joint plenary address [13], entered a new phase when I embarked on a new career in the University of California, Davis. Ironically our closer proximity terminated our collaboration. Lucien was engaged in writing his scholarly volume [14] and the removal of the necessity for one of us to travel to the other's university, which served to isolate us for a period of creative work, was removed. But, while our research collaboration ceased, our friendship developed, and Lucien taught me yet another skill, cross country skiing, an activity we still share when opportunity offers.

Lucien, scholar and friend, I salute you. I, in company with your many students who remain devoted to you, value you for your friendship, for what you have taught us, and for what you have contributed to the field. May you have many more years of creative work, and enjoyable skiing.

References

- [1] E. Polak. *Computational Methods in Optimization: a Unified Approach*. Academic Press, New York, 1971.
- [2] D. Q. Mayne and E. Polak. Feasible directions algorithms for optimization problems with equality and inequality constraints. *Math. Programming*, 11(1):67–80, 1976.
- [3] E. Polak and D. Q. Mayne. An algorithm for optimization problems with functional inequality constraints. *IEEE Transactions on Automatic Control*, 21(2), 1976.

- [4] D. Q. Mayne and E. Polak. An exact penalty function algorithm for control problems with control and terminal equality constraints—part 1. *Journal of Optimization Theory and Applications*, 32(2):211–246, 1980.
- [5] D. Q. Mayne and E. Polak. An exact penalty function algorithm for control problems with control and terminal equality constraints—part 2. *Journal of Optimization Theory and Applications*, 32(3):345–363, 1980.
- [6] D. Q. Mayne, E. Polak, and R. Trahan. An outer approximations algorithm for computer aided design problems. *Journal of Optimization Theory and Applications*, 28(3):231–252, 1979.
- [7] D. Q. Mayne and E. Polak. A superlinearly convergent algorithm for constrained optimization problems. *Mathematical Programming Studies*, 16:45–61, 1982.
- [8] D. Q. Mayne and E. Polak. On three approaches to the construction of non-differentiable optimization algorithms. In *Proceedings 11th IFIP Conference on System Modelling and Optimization*, Copenhagen, 1983. Springer Verlag.
- [9] D. Q. Mayne and E. Polak. Outer approximations algorithm for nondifferentiable optimization problems. *Journal of Optimization Theory and Applications*, 42(1):19–30, 1984.
- [10] E. Polak and D. Q. Mayne. On the solution of singular value inequalities over a continuum of frequencies. *IEEE Transactions on Automatic Control*, 26(3):690–695, 1981.
- [11] E. Polak, T. H. Yang, and D. Q. Mayne. A method of centers based on barrier functions for solving optimal control problems with continuum state and control constraints. In B. Wyman G. Conte, A. M. Perdon, editor, *New Trends in System Theory, Proc. Universita di Genova/Ohio State University Joint Conference*, pages 591–598, Genoa, Italy, 1991. Birkhauser.
- [12] E. Polak, J. E. Higgins, and D. Q. Mayne. A barrier function method for minimax problems. *Math. Programming*, 54(2):155–176, 1992.

- [13] D. Q. Mayne and E. Polak. Optimization based design and control of constrained dynamic systems. In *Proceedings of 12th IFAC World Congress on Automatic Control*, pages 129–138, Sydney, Australia, 1993. Plenary address.
- [14] E. Polak. *Optimization: Algorithms and Consistent Approximations*. Springer Verlag, New York, 1997. ISBN 0-387-94971-2.