

Book Review by G. Maier (published in Meccanica)

Undergraduate students and, in general, beginners in solid and structural mechanics are usually pleased to learn that the solutions of their linear elasticity problems are characterized by two extremum principles. Some celebrated professors of the past, like Arturo Danusso in Milan, suggested to see in this circumstance a universal tendency of nature, or of the divine Creator, towards optimality and some sort of symmetry in all phenomena. Such feelings may be strengthened later, when one learns that incremental (rate) solutions in classical elastoplasticity, “stable” in Drucker’s sense, are characterized by three pairs of dual extremum properties. People dealing with linear programming in operations research, economics, management, engineering, etc., know well that every optimization of linear function under linear inequality constraints uniquely defines a dual optimization of the same kind. Similar manifestations of harmony and orderly architecture of concepts in mathematical modelling of observable events become more and more fascinating as one proceeds deeper and deeper in various scientific fields. This is one of the main messages arising from David Gao’s book on duality in broad sense. In view of the real difficulty of a clear account of its contents, I take the liberty to merely cite here the titles of the chapters.

Part I Symmetry in Convex Systems

1. Mono-duality in static systems

2. Bi-duality in dynamical systems

Part II Symmetry Breaking: Triality Theory in Nonconvex Systems

3. Tri-duality in nonconvex systems

4. Multi-duality and classifications of general systems

Part III Duality in Canonical Systems

5. Duality in geometrically linear systems

6. Duality in finite deformation systems

7. Applications, open problems and concluding remarks

Each chapter ends with a commentary section, primarily devoted to historical remarks and special applications.

The topics dealt with in three appendices are:

A. Duality in Linear Analysis

B. Linear Operators and Adjointness

C. Nonlinear Operators

As for the meaning and implications of bi-, tri- and multi-duality, it seems to me preferable, and more prudent, to refer to the book rather than to try explanations in this review.

The style adopted by the Author is consistent with the approach deliberately adopted: pure, abstract mathematics; rigour in statements and proofs; rich symbology introduced once for all; no indulgence in computational aspects or real-life problems.

As a consequence, the book turns out to be tough reading for engineering students and researchers. However, in my opinion, it is an excellent, impressive and appealing book, perhaps in a sense unique, as far as I know. In fact, the volume provides a unifying survey of little known, but fascinating and potentially useful, aspects and properties of mathematical models in diverse areas of sciences (primarily, but not exclusively, mechanical sciences); it contains several novelties and numerous very recent research results; finally, various passages give evidence that it was written with enthusiasm, emotional participation in advanced research and rare width and breadth of scientific and humanistic horizons.

“Through pure mathematical analysis the intrinsic inner beauty in physical natural

can be revealed.” This is a quotation of a David Gao’s passage, but the book is enriched by many meaningful citations of thinkers in all times and continents, from mystic philosophers of ancient China (fourth century BC) to Einstein, Hilbert, Truesdell and other leaders of modern sciences.

The following quotation appeared in a recent issue of this Journal: “The preface is the most important part of the book. Even reviewers read a preface.” Both statements hold partly true for this book. The latter might be related to the difficulty (but also the appeal) of reading its chapters for an engineering-oriented reader like the present reviewer. The former reflects the unusual depth and philosophical flavour of the thoughts expressed there (e.g. “duality in nature is amazingly beautiful”, “simply mysterious, for it is the way nature was created”) and the two pages of acknowledgements (including the touching ones to the Author’s prematurely lost wife and brother) which accompany the Preface.

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