Recent Progress in Nonlinear Integer Programming

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Many decision making problems in real applications naturally result in optimization formulations in a form of nonlinear integer programming. Exemplary areas include capital budgeting, production planning, capacity planning, reliability networks and chemical engineering process. Although the past few decades have witnessed tremendous efforts in achieving theoretical and methodological development of (linear) integer programming, nonlinear integer programming has not received enthusiastic attention, partly due to its challenging feature resulted from a combination of combinatorial nature and the nonlinearity inherent in the problem. Existing solution methods for solving nonlinear programming problems have been mainly branch-and-bound methods and dynamic programming methods or their combinations [1-5]. Computational experiments have showed that these methods are only capable of handling small size problems.

Our main research goal has been to establish convergent duality theory and to develop efficient solution algorithms for large-scale nonlinear integer programming problems. The fundamental target underlying our theoretical development is to eliminate duality gap in the classical Lagrangian dual formulation. We have developed nonlinear Lagrangian theory that has yielded several new dual formulations with asymptotic zero duality gap [6-8]. The key concept is the construction of a nonlinear support for a nonconvex piecewise-constant perturbation function. Our numerical implementation of a duality-gap reduction process relies on some novel "cutting" procedures. Performing objective-level cut, objective contour cut or domain cut reshapes the perturbation function, thus exposing eventually an optimal solution to the convex hull of a revised perturbation function and guaranteeing a zero duality gap for a convergent Lagrangian method. We have successfully implemented convergent Lagrangian algorithms for nonlinear knapsack problems [9], nonlinear separable integer programming problems [10] and concave knapsack problems [11]. Extensive computational experiment has been carried out. The numerical results show that our proposed methods are capable of solving large-scale problems with up to several thousand integer variables in reasonable computation time. An internet-based computing interface is available for test problem solving (http://www.se.cuhk.edu.hk/~nlip).

The convexity of a function could be hidden in an original representation space. Convexification methods [14][15] have been proposed to transform a monotone function into an equivalent convex function in a new representation space. Based on the developed convexification transformation schemes and concave minimization techniques in global optimization, we have developed a branch-and-bound method for monotone nonlinear integer programming problems that often arise in complex reliability networks [12] and allocation resource problems [13]. Successful applications have been observed in optimization of reliability networks [16]. The convexification algorithm can find an exact solution of optimization problems in complex networks with the largest size ever reported in the literature.

For whatever definition of a local minimum in integer programming, there often exist multiple minima in integer programming problems, even in linear or strictly convex integer programming problems. In this sense, solving an integer programming problem is equivalent to searching for the global minimum among local minima. A global-descent method enables us to “escape” from the current local minimum to a better local minimum. By utilizing the concept of the filled function in continuous global optimization, we have recently developed a local-search and global-descent method ([17][18]) for general nonlinear integer programming problems without any special structure. Preliminary computational results are very promising with problem size up to several hundred integer variables. An internet-based computing interface is available for test problem solving (http://www.se.cuhk.edu.hk/~nlip).

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