GUIDO STAMPACCHIA

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1. Scientific education and first research activity at the Scuola Normale Superiore in Pisa and at the University of Naples.

Guido Stampacchia was born on March 26th, 1922 in Naples, to the family of Emanuele Stampacchia and Giulia Campagnano. Giulia belonged to a Jewish family of Florentine origin[1] that owned a factory which made hand embroidered household linen and linen garments for women. The Stampacchia family had origins in Lecce and practiced the Valdese Christian religion: the father, Emanuele, managed an iron tools factory which he was forced to sell off at the time of the war in Ethiopia, as a result of his refusal to join the fascist party. The young Guido got essentially a lay education, although as a child he attended the Valdese church together with his two sisters. He obtained his high school certification in classical subjects (maturità classica) at the age of eighteen from the Liceo-Ginnasio Gian Battista Vico in Naples, obtaining the excellent mark of 9/10 only in Mathematics and Physics. In spite of the classical studies he had followed, he intended to dedicate himself to mathematics and hence broadened his preparation in Mathematics and Physics by studying "the basic principles of the program prescribed for the scientific high school, seeking .... to find a logical process".[2]

In the autumn of 1940 he was admitted as an internal alumnus to the Scuola Normale Superiore of Pisa, for the undergraduate course in Pure Mathematics of the Classe di Scienze (Science Faculty), having secured the fifth position in the competitive entrance examination[3]; after that he completed brilliantly all the examinations in the curriculum of the first three years as required for the students of the Scuola Normale. In particular, he had Francesco Cecioni and Salvatore Cherubino among his teachers in the first three years of his university career, while in the third year he followed the courses of Leonida Tonelli on Analisi superiore (Advanced Analysis) and of Lamberto Cesari on Teoria delle funzioni (Theory of Functions). The latter, having graduated in 1933 under the supervision of Tonelli, was a professor in charge of a course from 1938, while Tonelli, the undisputed master of the Mathematical School in Pisa, taught the courses of Analisi infinitesimale and Analisi superiore, and maintained the chair of Analisi superiore during the three years – from November 1939 to October 1942 – when he had moved to the University of Rome. The advanced courses of Tonelli were concerned with trigonometric series, integral equations and calculus of variations, in a three-year cycle; when Stampacchia attended it, the course on Analisi

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[1] In some documents of the twenty years of fascism, the surname Campagnano was changed into Campagna, probably because of the then existing racist reasons.

[2] G. Stampacchia, *Nota sugli studi e le tendenze personali*, sent along with the application for admission to the Scuola Normale of Pisa, conserved in the personal files of G. Stampacchia as a student, in the Archives of the Scuola.

[3] The topic of the written test in Analysis, for the competitive examination of the Scuola, was the theory of real numbers.

superiore was dedicated to calculus of variations. During this period, the assistants of Tonelli at the University were Jaures Cecconi, who then became a professor at Genova, Landolino Giuliano, who later became a professor at the Naval Academy at Livorno, and Emilio Baiada who, as a professor, later taught at Palermo and Modena.

The courses at the Scuola Normale were organized by Tonelli and Giuliano. Tonelli used to organize two seminars, one for first level students and one for advanced students: respectively, Esercitazioni di Analisi e Geometria (Tutorial Sessions in Analysis and in Geometry) and Conferenze di teoria delle funzioni (Conferences in the Theory of Functions). He was the editor-in-chief of the Annali della Scuola Normale Superiore; later he was also the director of the Scuola Normale, during the academic year 1943-44, contributing greatly to the survival of the Scuola in such a critical and difficult moment for Pisa and the whole country. Stampacchia followed his tutorial course of Analysis and Geometry in 1941-42, took the examination in Theory of Functions the following year and, in 1943, passed the discussion on Ordinary Differential Equations, obtaining the mention of excellent preparation and excellent aptitude. Giuliano taught the courses of Complementary Mathematics I and II attended by Stampacchia, who took the relative examinations in his first and second year. During his stay at the Scuola Normale, Stampacchia had among his fellow students Giuseppe Colombo, Mario Dolcher, Jacopo Barsotti, who had joined the Scuola before him, Enrico Magenes, Roberto Conti, who joined a year later, and, finally, Aldo Andreotti, who was admitted in October 1942.

On the 24th of March 1943, Stampacchia, who had been drafted, informed the Administration of the Scuola Normale that he was expected to report at the Regia Aeronautica (the Royal Air Command) by the 28th of that month; anyway, he managed to take the examinations at the University of Pisa during the summer session of June 1943, securing marks of 30/30 *cum laude* from both Cesari and Tonelli. At the end of June he was sent to Rome to attend a course for Sergeant Cadet in technical specializations. He remained there until the 8th of September 1943, when he joined the Resistance Movement against the Germans, in the defense of Rome. After an adventurous trip, he regrouped with his parents and sisters in Isernia and returned to Naples only after the liberation of the city. He was assigned by the Liberation Army to administrative duties, and eventually discharged in June 1945.

In the meantime, taking advantage of a special Ministerial decree due to the war, he completed his fourth year of studies at the University of Naples. There he graduated, on the 28th of November 1944, obtaining his Laurea, on behalf of the University of Pisa, with the mark of 110/110 *cum laude*, discussing a thesis on ordinary differential equations, written under the guidance of Renato Caccioppoli. His thesis[4] was concerned with an adaptation of an approximation procedure for Volterra integral equations due to Tonelli[5] to boundary value problems for systems of ordinary differential equations.[6]

In the fall of 1944 he won a scholarship from the University of Naples for new graduates in mathematics, which allowed him to continue his studies under the direction of Renato Caccioppoli and Carlo Miranda; in addition, he worked as a voluntary assistant to the chair of Analisi Matematica (Mathematical Analysis) where he did tutorial work for the course of Analisi Algebrica (Algebraic Analysis). Meanwhile he also prepared himself for the Esame di Licenza (Final Examination) in Analisi Superiore at the Scuola Normale, which he passed with 70/70 *cum laude* on November 19th, 1945. As the topic of his examination, he had chosen the semicontinuity of the double integral in the calculus of variations $\iint_D f(x,y,z,s)dxdy$, with the integrand depending on the mixed second-order derivatives of the unknown function. Thus, from his very beginning as a

[4] Stampacchia 1947, *Ma* [1], p. 418. Here and in the following *Ma*[·] is used to indicate the numbering of the scientific publications of G. Stampacchia as introduced by Magenes 1978b.
[5] See G. Sansone, *Equazioni differenziali nel campo reale*, v. 1, pp. 45-48.
[6] Stampacchia 1947, *Ma*[1], pp. 413-414.

researcher, Stampacchia's interest for a topic very much studied by Tonelli and his school was evident, a topic on which he was to obtain very significant results within a span of few years.

In the academic year 1945-46, he shared with Jacopo Barsotti the first place for a position of Perfezionamento (Specialization) at the Scuola Normale in the Faculty of Sciences, but he declined, in order to accept a position of assistant at the Naval Institute at Naples, where Caccioppoli and Miranda were in charge of the courses in Analysis. A reason behind this decision, in addition to personal ones, was a certain dissatisfaction with his studies, which is so describes: "During the period I stayed in Pisa, I tried in several ways to develop a special interest in something, but I wandered among many different topics due to an absolute lack of a guide. Thereby, the reasons for the sacrifices I would have to make if I remained in Pisa became all the more meaningless."[7]

Stampacchia worked in Naples willingly and with satisfaction: having found in Caccioppoli a second teacher, he continued the studies in Differential Equations and Calculus of Variations he had begun with Tonelli. During the years 1945-46, 1946-47 and 1947-48 he fulfilled his teaching duties at the Naval Institute giving tutorial courses in Algebraic and Infinitesimal Analysis, as assistant in charge. At the same time, as a voluntary assistant, he helped with the courses on Analysis at the University, until he was given charge of the course entitled Istituzioni di Matematica (Principles of Mathematics) at the Faculty of Sciences, which he taught from November 1948 to December 1952. Moreover, in the academic year 1948-49, he held a CNR (National Research Council) scholarship to work on Calculus of Variations and methods of Functional Analysis at the Mathematics Institute of the University of Naples.

The scientific and economic independence he achieved in Naples permitted him to realize his desire, already evidenced when he gave up the specialization at the Scuola Normale, and marry Sara Naldini, his fellow student at the University. After the wedding was celebrated, in October 1948, the couple settled in Naples in the family house, where their children Mauro and Renata were born, in 1949 and 1951.

At the age of twenty seven, after having been declared eligible in a competition for the position of assistant in which he participated in November 1948, he was appointed as assistant with tenure to the chair of Mathematical Analysis at the University of Naples, on July 1st, 1949. Further, in the month of April 1951, he obtained the Libera Docenza (Habilitation) in Mathematical Analysis: for this examination, he presented 14 papers ranging from ordinary differential equations to the theory of functions of real variables, from calculus of variations to partial differential equations.[8]

In the field of ordinary differential equations, which was the topic of his graduation thesis, Stampacchia studied, generalizing a problem posed by Nicoletti at the end of the eighteenth century, the problem of determining the solutions of a first order system of nonlinear differential equations, in the case when the boundary conditions were also given in a nonlinear form (*Ma*[1], [5]). In the Piccole Note of Unione Matematica Italiana (UMI), he gave a condition under which the solution of an equation of order n depending on a parameter, together with conditions to identify a polynomial of degree n, is reduced to an ordinary boundary value problem for an equation of order $n+1$ (*Ma*[7]). Further, in a note appeared in the Rendiconti Lincei, he gave a functional interpretation of the Peano phenomenon concerning the lack of uniqueness in the Cauchy problem (*Ma*[9]). Stampacchia returned to the study of ordinary differential equations during the fifties, when he solved the problem of determining an integral curve for a first order system lying on an n

[7] G. Stampacchia to L. Russo, Naples 21-1-1946, conserved in the personal files of G. Stampacchia as a student, in the archives of the Scuola Normale. Enrico Magenes remembers with emotion a visit he made with Stampacchia, on the first days of November 1945, to the house in Asciano Pisano where Leonida Tonelli had retired for health reasons. On this occasion, Tonelli advised Stampacchia to leave Pisa and benefit the possibility of working with Miranda and especially with Caccioppoli, whose charm had strongly attracted Stampacchia (Magenes 2000, p. 27). Leonida Tonelli died after a few months on 12 March 1946.

[8] For details and an exhaustive examination of these papers, see Magenes 1978a, pp. 717-722 (XIII-XVIII).

dimensional manifold (*Ma*[21]); and when, in a lecture delivered at Catania in May 1956, he gave a survey of the theory of boundary value problems for systems of ordinary differential equations (*Ma*[26]).

Among his first research works are the study of the Goursat problem at the large for a second order nonlinear hyperbolic partial differential equation in two variables (*Ma*[10]) and, due to his closeness with Caccioppoli, the paper (*Ma*[2]), in which he proves, for rectifiable surfaces, a conjecture about the uniqueness of the definition of area which fulfils lower semicontinuity condition.

The work of Stampacchia in the Calculus of Variations, stimulated by the results obtained and the techniques developed by Tonelli, which he had rapidly mastered during his university career, continued with applications of the direct methods to double integrals depending on generic differential operators acting on the unknown function. In one of his earliest papers, semicontinuity was considered (*Ma*[3]); this work was completed by determining the conditions ensuring the existence of the minimum (*Ma*[11]). These results were the subject of a communication at the III UMI Congress, held in Pisa in September 1948.[9] In the case of functionals depending only on second order derivatives (*Ma*[4]), taking the Laplacian of the unknown function as the differential parameter, the minimization problem is set in the class of once continuously differentiable functions, which are assumed to be absolutely continuous along lines parallel to the axes (absolutely continuous according to Tonelli), and such that the pure second derivatives are integrable; for functionals depending on the mixed derivatives (*Ma* [6]), one considers a class of doubly absolutely continuous functions (absolutely continuous according to Vitali).

Studying the papers of Fubini and Beppo Levi on the minimum principle for the Dirichlet integral, Stampacchia realized[10] that, to prove the existence of a minimum for the case of double integrals, in the use of the direct method was not necessary to have uniform convergence; and that the appropriate function space for formulating the problem was not the space of absolutely continuous functions. Thus, with the encouragement of Caccioppoli, he began to examine functions separately continuous with respect to each variables together with the associated notion of quasi uniform convergence and the corresponding compactness criteria (*Ma*[8], [12]), which found applications in the calculus of variations (*Ma*[13]) and in the study of the Dirichlet problem for second order elliptic equations in two variables (*Ma*[14]). In particular, in the paper (*Ma*[13]) published in 1950 in the Giornale Battaglini, of which Caccioppoli and Miranda were the editing directors, in order to overcome the limitations of Tonelli's theory, Stampacchia introduced a class of functions of the type of Sobolev space, in the two-dimensional case. Thus, independently and using different methods, he found results similar to those of C.B. Morrey on variational problems for multiple integrals,[11] about which he learned much later because of the then existing scientific isolation of Italy, due to the war. Stampacchia remarks explicitly that, while his formulation was similar to Morrey's as far as the nature of the functions considered, the treatment was different in so much as Morrey makes use of functions defined up to sets of measure zero, thus precluding an analysis of boundary traces.

Naturally the existence theorems so obtained furnish minimizing functions in the new class, leaving open the regularity problem of the solutions thus found.

The depth and penetration with which these problems were treated and understood are very aptly described in the words with which Ennio De Giorgi remembers a visit to Naples during this

[9] G. Stampacchia, *Gli integrali doppi del calcolo delle variazioni in forma ordinaria*, Atti III Congresso UMI, 1948 (1951), p. 110.
[10] Stampacchia 1950, *Ma*[13], p. 171 – Stampacchia 1996-97, p. 31.
[11] C.B. Morrey, *Existence and differentiability theorems for the solution of variational problems for multiple integrals*, Bull. Amer. Math. Soc., v. 47, 1940, pp. 439-458.

period: "Picone was very much interested in problems of the type $\min_{\Omega}\left(\int_{\Omega} g + H_{n-1}(\partial\Omega)\right)$ and thought that Caccioppoli was the right person to find a path to the solution. Hence, he sent me for some days to Naples, where, talking to Caccioppoli, Stampacchia and Carlo Miranda, I could experience all the richness of their ideas from their live voices, much more than as those ideas were stated in their very ingenious writings. In the words of Caccioppoli, Stampacchia and Miranda their personal experiences and the teaching of their masters Picone and Tonelli were integrated, and the spirit of the direct methods of the calculus of variations and, in particular, of the procedure divided into four fundamental steps: relaxation, semicontinuity theorems, representation theorems, regularity theorems, came out clearly."[12]

The years that Stampacchia spent in Pisa and Naples characterize the formation of his personality as an analyst: he was a passionate specialist in calculus of variations and in the theory of partial differential equations, a practitioner and an inspirer of research works of considerable depth and originality of thought. As is well known, his work has contributed notably to the progress of mathematics and the fields of research opened by him are still drawing the attention of the international mathematical community.

2. The chair at the University of Genova and the admittance into the international mathematical community.

In 1952 Stampacchia came out first in the national competition for a chair of Algebraic and Infinitesimal Mathematical Analysis at the University of Palermo[13]; he was nominated Professor on Probation (Professore Straordinario) in the Faculty of Mathematical, Physical and Natural Sciences of the University of Genoa on the 15th of December 1952. In December 1955, he was promoted Full Professor (professore ordinario).[14] He then settled himself with his family in Genova, where his daughters Giulia, in 1955, and Franca, in 1956, were born.

In the years between his habilitation and full professorship, Stampacchia[15] generalized to the case of *n* variables the class of functions he had introduced for the two dimensional case, and he proved the corresponding compactness criteria (*Ma*[15]). He presented these results at the Congress of UMI held at Taormina in October 1951 (*Ma*[16]). In his communication, he also treated the question of existence of the minimum for multiple integrals depending on the first and second derivatives of the unknown function, leading to the Euler equation being satisfied almost everywhere and thus strengthening the relation between calculus of variations and partial differential equations, a constant theme of all his research. The minimum is attained (*Ma*[17]) in a class of functions having traces on the boundary together with their normal derivative; since it is shown to satisfy the Euler equation, one obtains existence results for a fourth order elliptic equation with assigned boundary values of both the function and its normal derivative. The study of these boundary conditions led Stampacchia to examine the question of the approximability of a function on an assigned surface (*Ma*[18]).

As to the differentiability properties of the minimizing function of multiple integrals depending on the gradient (*Ma*[19]), we recall Stampacchia's results of local integrability of the

[12] De Giorgi 1985, p. 185.
[13] The report of the board of juries, constituted by L. Fantappié, G. Scorza, C. Miranda, S. Cinquini and L. Amerio, is published in Bollettino ufficiale M.P.I., II part, 9 July 1953, n. 28, pp. 2192-2203.
[14] The report of the board of juries, constituted by F. Cecioni, C. Miranda and S. Cinquini, is published in Bollettino ufficiale M.P.I., II part, 26 July 1956, n. 30, pp. 5071-5073.
[15] See Magenes 1978a, pp. 722-725 (XVIII-XXI).

second derivatives[16] and of analyticity of solutions, with Hölder-continuous first order derivatives, of regular problems. These regularity results, together with an analysis of variational problems for multiple integrals depending on the first order derivatives of the unknown function, were illustrated in a lecture delivered in Torino in January 1954 (*Ma*[20]). The next step was to study the Euler equations by the direct methods of the calculus of variations in the case of Neumann or mixed boundary conditions (*Ma*[23]). Some of the results of this paper, dedicated to Mauro Picone on the occasion of his seventieth birthday, are the subject of a communication presented at the International Congress of Mathematicians held in Amsterdam in September 1954 (*Ma*[22]), while some remarks on the existence and uniqueness of solutions were published in Rendiconti dell'Accademia delle Scienze of Naples (*Ma*[24]).

The variational method was also used by Stampacchia to treat the so called transmission problem, namely, the problem of two equations in two domains having parts of their boundary in common and with a natural condition of matching on the common part of the boundary (*Ma*[27]). This kind of a situation arises in the study of phenomena taking place in a stratified medium. The leading idea is again solving the problem in a weak form, then studying the regularity of the weak solution thus found. A clear exposition of the results on transmission problems for elliptic equations was presented in a talk at the Seminario Matematico of Bari in December 1960 (*Ma*[32]).

Stampacchia's interest in the different classes of functions used in proving existence in weak form can be found in his lectures (*Ma*[28]) at the CIME course on Singular Integrals and Related Questions, held at Varenna in June 1957, where he presented a theory of completion of function spaces following ideas suggested by Aronszajn and Smith.[17]

During the years he spent in Genoa, Stampacchia taught courses in Analisi Matematica, Analisi Superiore, Matematica Superiore (Higher Mathematics), Istituzioni di Matematica II, and Topologia. In addition, during the academic year 1960-61, when he had already moved to Pisa, he also taught a course of Complementi di Matematica (Complements of Mathematics) at the School of Engineering. In February 1960, he was appointed to represent the Faculty of Sciences in the Council of Directors of the newly created Centro di Calcolo Numerico.

He interacted agreeably with his colleagues Eugenio G. Togliatti, Enzo Martinelli and Francesco Sbrana. When Martinelli moved to Rome, he favoured the arrival of Enrico Magenes, with whom he had always kept a close relation. He established scientific and friendly relations also with Jaures P. Cecconi, Giulio Aruffo and Emilio Gagliardo; later on, Sergio Campanato also worked with him. During these years, he began to entertain the intense international relations, which were to characterize his future mathematical career. In August 1954, at the conference "Problemi esistenziali e qualitativi per le equazioni differenziali lineari alle derivate parziali" organized by Gaetano Fichera in Trieste, Stampacchia met Louis Nirenberg, with whom he remained a very close friend until his death. Together with Magenes, he had guests as prestigious as Laurent Schwartz, Henri George Garnir, Antoni Zygmund, Bernard Malgrange and Nachman Aronszajn; he first met Jacques-Louis Lions at Nice, in September 1957, during the Réunion des Mathématiciens d'Expression Latine.

He very often traveled abroad for scientific studies and collaborations. His travels were so frequent that it is impossible to remember all the visits he made, either to attend conferences or to give talks, to innumerable research institutes in Italy and abroad. We shall limit ourselves to mention long-time visits, and only to the most prestigious institutes. Quite often, these visits resulted in significant scientific publications.

As a result of common interests and continued collaboration with Magenes, an exposition of a complete and general survey of the different approaches to boundary value problems for linear

[16] This result was obtained by extending the methods introduced by C. Miranda, *Sui sistemi di tipo ellittico di equazioni lineari a derivate parziali del primo ordine in n variabili indipendenti*, Mem. Accad. Naz. Lincei, s. 8, v. 3, 1952, pp. 85-121.
[17] N. Aronszajn - K.T. Smith, *Functional spaces and functional completion*, Ann. Inst. Fourier, 6, 1956, pp. 125-185.

elliptic differential equations of arbitrary order was written up in the spring of 1958 (*Ma*[29]). Even more than ten years after its publication, and in spite of notable further developments on the subject matter, this article remained a remarkable instrument for all those who wished to dedicate their studies to these issues. The subject was reconsidered later, in the general lecture (*Ma*[31]) Stampacchia gave at the VI UMI Congress, held in Naples in 1959.

During the mid fifties, the work of Stampacchia on the differentiability properties of the extremals of regular multiple integrals was concerned with different types of questions: on the one hand, existence of square integrable second order derivatives of the solutions to minimum problems, solutions being a priori of very low regularity; on the other hand, analyticity in the case of Hölder continuous first order derivatives (*Ma*[19], [25]). A relation between these results was missing, namely, to assure the analyticity of the minimizing functions whenever the integrand in the regular integral is analytic. Stampacchia stimulated De Giorgi's thoughts on this important problem,[18] which De Giorgi solved in a famous paper presented at the V UMI Congress held in Pavia in October 1955, and published later on, in 1957.[19]

Continuing along the De Giorgi's ideas, Stampacchia considered linear elliptic equations with discontinuous coefficients in spaces of dimension $n > 2$; in the span of about ten years, he obtained results in various directions: L^p estimates for the solutions, the maximum principle, Hölder continuity of the solutions up to the boundary, existence and properties of the Green's function for the Dirichlet problem, characterization of regular points on the boundary.[20] The notable body of scientific publications of the highest international standards on elliptic equations, to which these works of Stampacchia belong, is very well illustrated by Carlo Miranda in a lecture at the VIII UMI Congress,[21] in whose reference list the name of Stampacchia is found among the most eminent specialists in the field.

He first obtained preliminary regularity results (*Ma*[30]), such as boundedness and integrability of solutions for a large class of boundary value problems for second order elliptic equations with bounded and measurable coefficients, imposing only the "cone condition" on the boundary of the domain. In order to explain his technique, he recalled the idea of De Giorgi to obtain the interior regularity of the extremals. Moreover, in the appendix, by comparing some results from the theory of capacity to the Sobolev inequality, he glimpsed a relation between these results and some isoperimetric inequalities. The questions of summability and of boundedness were taken up again in July 1960 (*Ma*[35]), in a lecture at the International Symposium on Linear Spaces held in Jerusalem, refining the technique of truncation with a lemma, by now classical, on decreasing functions. Some limiting cases of summability of the known right hand side were analysed later in (*Ma*[39]), during his visit at the Courant Institute of Mathematical Sciences.

The Hölder continuity up to the boundary of the solution was essentially considered in a memoir dedicated to Giovanni Sansone in the occasion of his seventieth birthday (*Ma*[34]), which had been preannounced by a Comptes Rendus note in February 1960.[22] More precisely, for equations more general than those considered by De Giorgi, and for an "admissible" class of sets Ω introduced to this purpose, Stampacchia proves Hölder continuity in $\overline{\Omega}$ of the solutions of the Dirichelet problem, of the Neumann problem, and of the mixed problem in which the boundary data are discontinuous, but the boundary of Ω, the coefficients and the right hand side of the equation are

[18] On this question, in addition to the work of De Giorgi, we recall those of J. Nash and, later, of J. Moser.

[19] E. De Giorgi, *Sulla differenziabilità e la analiticità delle estremali degli integrali multipli regolari*, Mem. Accad. Sci. Torino, s. 3, v. 3, 1957, pp. 25-43. A pre-publication note was published in 1956 in Rendiconti dell'Accademia dei Lincei.

[20] See Magenes 1978a, pp. 726-732 (XXII-XXVIII).

[21] C. Miranda, *Progressi e orientamenti della teoria delle equazioni ellittiche negli ultimi quindici anni*, Atti VIII Congresso UMI, 1967 (1968), pp. 23-54.

[22] G. Stampacchia, *Solutions continues de problèmes aux limites elliptiques à données discontinues*. C. R. Acad. Sci. Paris, 250, 1960, pp. 1426-1427.

regular. The geometrical conditions ensuring "admissibility" of a set Ω were exposed at the Colloque sur l'Analyse Fonctionnel, held at Louvain in July 1960 (*Ma*[33]).

In 1960 Stampacchia was a visiting professor for one month at the Institut des Hautes Études in Paris, where he gave a brief course under the auspices of the Séminaire Schwartz, during the year devoted to partial differential equations and interpolation. In the published version of this course (*Ma*[36]), he obtained the integrability and the boundedness of solutions of second order elliptic equations with discontinuous data, under less restrictive hypotheses on lower order terms than those considered in earlier works (*Ma*[30], [35]). In the month of November of the same year, he sent a letter of support and solidarity, which was jointly signed by 46 other Italian mathematicians, to Laurent Schwartz, who had been removed from his chair at the École Politechnique.

During his academic career, Stampacchia served as a member in many committees for national competitions for professorial positions in Analysis. Among these, we mention that he was the secretary of the selection committee[23] for a position in Mathematical Analysis at the University of Messina, for which Ennio De Giorgi was first among the selected candidates.

Even after he moved to Genoa, he kept a privileged contact with Naples, both because of his close relationship with his sisters and his other relatives who lived in Naples and because of his relations with the scientific community in Naples, first of all with C. Miranda and Caccioppoli, and also with Federico Cafiero, Donato Greco and Renato Vinciguerra. He was elected a corresponding member of Società Nazionale di Scienze, Lettere ed Arti of Naples in November 1954.

As remarked by Magenes,[24] Stampacchia was always proud of his native Naples and of the family background in which he was born; in fact, he was very happy of being born in such a special place and in an unusual family. These peculiar roots were particularly suited to his personality, rather nonconformist, and certainly contributed to his formation. Part of his characteristic nature was that of a very amiable and easy-going gentlemanliness, a simplicity coupled with the consciousness of his important position in the mathematical community, a very strong sense of criticism and a great sense of humour. Very frank in expressing clearly whatever he thought, he had a very generous and free attitude towards friends and students which was reciprocated with affection. It is worth recalling the testimony of Haïm Brezis, who remembers the beginning of their long and fruitful mathematical relation: "Pendant la préparation de ma thèse j'eu la chance de rencontrer trois maîtres, Félix Browder, Louis Nirenberg et Guido Stampacchia, qui ont donné une ouverture internationale à mon travail. […] Ma première invitation mathématique est arrivée de l'Université de Pise où enseignait G. Stampacchia. J'avais à peine vingt-trois ans [1967] et j'ai des souvenirs merveilleux de cette visite d'un mois. L'hospitalité de G. Stampacchia était légendaire; il était impossibile de régler une addition, ni même de payer un café, en sa présence. Comme j'ai l'ai dit, mes connaissances en EPD (équations aux dérivées partielles) étaient très fragmentaires et je n'étais même pas familier avec le célèbre principe du maximum; plus précisément, il était enseigné à Paris – en théorie du potentiel – mais sous une forme tellement abstraite et déguisée que le lien avec les EPD s'était perdu. Au lieu d'être surpris de mes lacunes et de me suggérer des lectures, G. Stampacchia s'est chargé lui-même de me l'enseigner en suivant une approche très élégante[25] qu'il avait découverte."[26]

[23] The other members of the board were Giovanni Ricci, Carlo Miranda, Gianfranco Cimmino, and Sandro Faedo.
[24] Magenes 1978a, p. 715 (XI).
[25] See Stampacchia 1963, *Ma* [38], pp. 387-388 – Stampacchia 1996-97, pp. 399-400 and Stampacchia 1965, *Ma* [45], pp. 206-207 – Stampacchia 1996-97, pp. 488-489.
[26] J. Vauthier, *Haïm Brezis un mathématicien juif*, Beauchesne, Paris, 1999, p. 25. Similar feelings are also expressed by David Kinderlehrer (Kinderlehrer 1988).

3. **Back to Pisa: the golden age of the Mathematical Institute, the Feltrinelli prize and the Presidency of Unione Matematica Italiana**.

While Stampacchia was in Genoa, Sandro Faedo, who had returned definitively to Pisa to the chair left vacant at the death of Tonelli, had already started to do his best to strengthen the Istituto Matematico. Faedo began by bringing in Aldo Andreotti from Turin; together, they tried from 1956 to 1958 to convince Stampacchia to move from Genoa to Pisa, offering him first the chair of Cecioni and then a position at the Scuola Normale.[27] For various reasons, Stampacchia's transfer did not come through; however, the mathematical community in Pisa was enriched by the arrival of Edoardo Vesentini at the University and of Ennio De Giorgi at the Scuola Normale. Thus, in 1960 when a position in Mathematical Analysis at the University of Pisa was available, Stampacchia eventually accepted the offer and moved from Genoa to Pisa with his family. In the same year, Barsotti too returned to Pisa, as Giovanni Prodi and Sergio Campanato did a little later.

The presence of this group of highly active and qualified experts in different fields made the scientific atmosphere of the Institute of Mathematics very bright and also led to useful and intense joint collaborative research. Moreover, international relations became very frequent and constructive, with short and extended visits of mathematicians from Pisa to foreign institutions and also by way of the presence of very distinguished visitors from abroad. Among the guests of the famous office with red tapestry, reserved for distinguished visitors at the Institute – at that time located in "La Sapienza", the historical seat of the University of Pisa – and then of the new building at Via Derna, we recall Oscar Zarisky, Hans Lewy, Philip Hartman, Louis Nirenberg, Armand Borel, Joe J. Kohn, Shmuel Agmon, Bernard Malgrange, Donald G. Aronson, Pierre Grisvard, Stanley Kaplan, and Robert Seeley. Thus, the Institute of Mathematics at Pisa became internationally renowned and a pole of attraction and training for research workers both Italian and from abroad, among which Enrico Bombieri stands.

From September 1961 to September 1963, Stampacchia moved with his family to the United States as a Temporary Member of the Courant Institute of Mathematical Sciences of New York University. On this occasion he also visited many other institutions and universities in the United States; in particular, at the University of Minnesota at Minneapolis he established scientific relationships with Walter Littman, Hans Weinberger and James Serrin. In the month of August 1962 he was one of the invited speakers at the International Congress of Mathematicians held in Stockholm; in his talk (*Ma*[37]) he presented the main results obtained up to that time on second order elliptic equations in divergence form with bounded measurable coefficients.

The papers published during his long stay in America are of great relevance. Stampacchia, in collaboration with Littman and Weinberger, obtained a very refined result on regular boundary points for the Dirichlet problem associated to a uniformly elliptic operator with discontinuous coefficients, and he was led to a detailed analysis of the properties of the Green's function and of the capacitory potentials (*Ma*[40]). In March 1963 he completed a paper (*Ma*[38]) strictly related to Calculus of Variations. In this paper[28] he obtained, under suitable hypotheses on the boundary of the domain and on the boundary condition, the existence and regularity of the minimizers of integrals which are only regular or uniformly regular, according to the kind of dependence on the unknown function. These results were obtained by the use of the maximum principle, proved by the truncation method; they can be applied to the problem of minimal surfaces, which could not be treated either by the already mentioned theorem of De Giorgi or by its extensions due to C.B. Morrey and O.A. Ladyzenskaja and N.N. Ura'ltseva.

[27] After the death of Caccioppoli , Stampacchia was contacted for transfer also from the University of Naples.
[28] See Magenes 1978a, pp. 734-735 (XXX-XXXI).

A new line of research began at this stage,[29] to which S. Campanato and G.N. Meyers had already made contributions: namely, interpolation between $^*L^{p,\lambda}$ spaces – which contain for particular values of the parameter λ the Lebesgue spaces of measurable functions whose powers are integrable and the spaces of Hölder continuous functions – and their application to elliptic equations (*Ma*[41]). These results were presented at the VII Congress of UMI held in Genoa at the end of September and October 1963 (*Ma*[42]), while the Dirichlet problem, in the limit case of the space of bounded mean oscillation (BMO) functions of John and Nirenberg, was considered in a section of *Ma*[39].

In the Conference "Convegno Lagrangiano", promoted by the Accademia delle Scienze di Torino in October 1963 in the occasion of the one hundred and fiftieth anniversary of the death of Lagrange, Stampacchia presented a survey (*Ma*[46]) of the main developments of the principle of minimum in the calculus of variations or, more precisely, of the relation between the minima of regular multiple integrals and the boundary value problems for elliptic partial differential equations, thus putting in a historical perspective the most recent results, from Sobolev spaces to trace theorems, from the maximum principle to the regularity of weak solutions.

After returning to Pisa from the United States, Stampacchia continued his study on the interpolation between function spaces, refining, among others, certain properties of inclusion between Morrey spaces (*Ma*[47]). As an application of these results, he and Campanato obtained in a joint paper some L^p estimates for the derivatives of solutions of elliptic equations (*Ma*[50]). Finally, in his talk at the Conference Equadiff II, Differential equations and their applications, held in September 1966 in Bratislava (*Ma*[54]), he presented a panoramic survey of the $^*L^{p,\lambda}$ spaces and of their use in interpolation theory and in elliptic equations.

As we have already observed, in his study of variational equations Stampacchia often considered the analysis of potential and capacity theories together; his well known generalization of Lax–Milgram lemma on coercive bilinear forms to convex sets can be put in this context (*Ma*[43]). Thus, the theory of variational inequalities was born, driven by the solution given by Gaetano Fichera to the Signorini problem on the elastic equilibrium of a body under unilateral constraints[30] and by Stampacchia's work on defining the capacitory potential associated to a non symmetric bilinear form.

In the spring of 1964 he was a visiting professor for a month at Collège de France, on an invitation by Jean Leray. He presented his work on interpolation spaces and he gave a course on second order elliptic equations in divergence form with bounded measurable coefficients, under the auspices of Séminaire Leray sur les èquations aux dérivèes partielles. In the published paper based on this course (*Ma*[44]), well known results for the case of the Laplace equation, such as the maximum principle and the properties of the Green function for the Dirichlet problem, were extended to second order elliptic equations in divergence form with bounded measurable coefficients. Also the extension of Harnack inequality by J. Moser,[31] based on the important result of John and Nirenberg on bounded mean oscillation functions, was proved. These topics can be found in an extensive paper published in Annales de l'Institut Fourier in 1965, a paper which was especially devoted to the Dirichlet problem for an elliptic operator in divergence form having discontinuous coefficients and lower order terms (Ma[45]). In this paper, among other things, Stampacchia made use of his result on non symmetric coercive forms to show the existence of the capacitory measure and of the capacitory potential, and to obtain a comparison of the capacities corresponding to different operators. The theory of elliptic equations with divergence structure and

[29] See Magenes 1978a, pp. 733-734 (XXIX-XXX).

[30] G. Fichera, *Problemi elastostatici con vincoli unilaterali: il problema di Signorini con ambigue condizioni al contorno*, Mem. Accad. Naz. Lincei, s. 8, v. 7, 1964, pp. 91-140. A preliminary note was published in Rendiconti dell'Accademia Nazionale dei Lincei in 1963.

[31] J. Moser, *On Harnack's theorem for elliptic differential equation*, Comm. Pure Appl. Math., v. 14, (1961), pp. 577-591.

with discontinuous coefficients was reviewed in the summer course he gave in the Séminaire de Mathématiques Supérieures at the University of Montreal. A presentation of this topic in its most general form is found in the notes of these lectures published in book form (*Ma*[52]).

Many times at the University of Pisa, Stampacchia was entrusted, in addition to courses in Analysis, with the courses of Analisi Superiore (Higher Analysis), Metodi Matematici della Fisica (Mathematical Methods in Physics) and Calcoli Numerici e Teoria dei Grafi (Numerical Calculus and Graph Theory); he also taught courses in Equazioni Differenziali and Analisi Superiore at the Scuola Normale. In November 1966, he was elected as the director of the Istituto di Matematica.

At the Institute, in addition to his already mentioned collaboration with Campanato, Stampacchia worked with other colleagues, interested in fields other than Analysis. An important aspect of that period was the weekly Seminar, which all the members of the Institute used to attend independent of their specific research field. In this context, we recall the collaboration with Andreotti and Vesentini on Carleman estimates for the Laplace-Beltrami equation on complex manifolds. Stampacchia contributed significantly to this work proving an inequality "which highlights the crucial role of the completeness of the metric of the manifold."[32]

Stampacchia and De Giorgi were able to put in evidence a property of minimal surfaces which was not encountered in the case of solutions of elliptic equations in general. Extending a result of Lipman Bers in two dimensions, they showed that a minimal surface which can be represented in Cartesian form on an open set in $\mathfrak{R}^n$ can have singularities in a compact set of zero capacity of order 1 or, equivalently, the singularity set can be a compact set of zero (n-1)-dimensional Hausdorff measure (*Ma*[48]).

In collaboration with M.K. Venkatesha Murthy of Tata Institute of Fundamental Research in Bombay, who was a visitor to Pisa several times starting from 1963, Stampacchia considered degenerate elliptic operators, that is, those operators for which the so called ellipticity constant is replaced by a function. This means that the ellipticity constant can depend on the point in the domain and can also vanish on some subset of points, but this function together with its reciprocal satisfies suitable integrability conditions. This leads to the consideration of differential equations in the context of Sobolev spaces with weights and hence to the necessity of recovering in such spaces the relevant properties which allow one to obtain the results known for boundary value problems associated to elliptic equations with discontinuous coefficients (*Ma*[56]).

The results in the papers with De Giorgi[33] and with Murthy (*Ma*[55]) were presented in the Conference "Le equazioni alle derivate parziali", held at Nervi (Genova) in February 1965, of which Stampacchia was one of the organizers.

While Stampacchia was in Chicago as a visiting professor in May 1966, on a proposal of Giovanni Sansone, he was awarded the Feltrinelli Prize for Mathematics, Mechanics and Applications of the Accademia Nazionale dei Lincei. In the motivation for awarding the prize, the committee[34] reviewed the important research activity of Stampacchia and underlined "the vast and ample scientific production", "the importance of the results" and "the high esteem and position that these have secured him in the international field".[35] He was appointed as a Corresponding Member of the Accademia dei Lincei in July 1968 with similar motivations. In fact, his frequent visits abroad, his extensive scientific relations in the international field and his vast group of students, direct or indirect, both Italians and foreigners, contributed very much to broaden the horizon of

[32] Vesentini 1980, p. 12.

[33] E. De Giorgi - G. Stampacchia, *Sulle singolarità eliminabili delle ipersuperficie minimali*, Atti del Convegno su Le Equazioni alle Derivate Parziali (Nervi, 1965), Edizioni Cremonese, Roma, 1966, pp. 55-58.

[34] The members of the committee were Beniamino Segre, Mauro Picone, Enrico Pistolesi, Giovanni Sansone, Alessandro Terracini and Bruno Finzi.

[35] *Relazione per il conferimento del premio "Antonio Feltrinelli"*, Rendiconti delle adunanze solenni, v. VII (1965-1976), Atti Accad. Naz. Lincei 1976 (1977), p. 143.

studies and research activities and to enhance the prestige of the Italian School of Mathematics in the scientific world.

The intense scientific activity of those years did not divert Stampacchia's attention from the problems of the teaching of Mathematics and of the formation of teachers, and he personally undertook the work of channeling young people towards mathematical studies. In an article, originally published in Bollettino della Società ex-alunni della Scuola Normale and later reproduced also in the Bollettino dell'UMI,[36] he analysed the genesis of the organization of teaching and of the university curriculum of studies, and made a proposal to reorganize the teaching scheme, including also a three year plan for the future teachers of middle schools. Moreover, he accepted to participate in a pre-University orientation course organized at Erice by the Scuola Normale in September 1966, where he delivered some lectures on the subject "Mathematics as research and as an instrument of scientific enquiry and technique". Stampacchia considered mathematics and, in particular, Mathematical Analysis as a fundamental instrument to study natural phenomena. Like Tonelli, he used to remark that the evolution of a phenomenon is often governed by principles which correspond to a maximum or a minimum of some integral. Accordingly, he assigned a prominent role to Calculus of Variations and to the Theory of Differential Equations in the understanding of problems in Physics and hence in establishing a close relationship between theory and applications. In the preliminary notes of his Erice lectures, one finds remarks expressing his view of Mathematics and his unified vision of theoretical and applied research: "Mathematics, as an expression of human thought, reflects the active will, the contemplative reasoning, the desire for aesthetic perfection. Its fundamental basis consists of logic and intuition, analysis and construction, generalities and individualities. Any development of Mathematics has without doubt its psychological origins in more or less practical requirements or demands, but once it is initiated under circumstances of necessity it acquires a value by itself and transcends the limits of its immediate utility. This trend from the applied towards the theoretical science shows itself continuously in history."

Stampacchia appreciated in Mathematics its utility in addition to its intrinsic beauty. He believed that a dynamic relationship between theory and application was of fundamental importance for the development of Mathematics. Quite often, "a theory in Pure Mathematics may become very useful in Applied Mathematics and, conversely, problems suggested by Applied Mathematics can lead to a new theory in Pure Mathematics."[37] He further stated: "In the last three decades Mathematics has gone through a period of profound critical re-examination, trying to recognize its fundamental structure in an abstract manner; at the same time, an increasing number of applied sciences discovered in Mathematics a basic instrument and indicated to it new fields of enquiry. Never before as in this period, has one seen this process of interaction by which, on the one hand Mathematics creates new instruments and new languages for applied sciences, and on the other the latter sciences, with their specific problems, give rise to new areas of research in Mathematics which were unthought of before. Mathematics with its various aspects thus gets inserted as a fundamental fact in cultural, scientific and technical development."[38]

In 1967 Stampacchia was elected, with 239 votes, President of the Unione Matematica Italiana (UMI), of which he was a member since 1948 and a member of the Scientific Committee from 1964. He remained in this office till 1973, when he was re-elected, as per his wish, a member of the Scientific Committee.

As President, he gave the opening addresses at the VIII and at the IX Congresses.[39] In these addresses he identified the role of UMI as that of "preservation and improvement of the level of

[36] G. Stampacchia, *Note sull'insegnamento della matematica*, Boll. UMI, s. 3, v. 21, 1966, pp. 186-190.
[37] G. Stampacchia, *Matematica pura e applicata negli sviluppi attuali*, introductory report to the Congress "Rapporti tra ricerca matematica pura ed applicata in Italia", Siena, 27-29 September 1973.
[38] G. Stampacchia, *Discorso*, Atti VIII Congresso UMI, 1967 (1968), p. 19.
[39] The VIII Congress of UMI was held in Trieste from 2 to 7 October 1967. The IX Congress of UMI was held in Bari from 27 September to 3 October 1971.

mathematical research in Italy and of modernization of the teaching"[40] and he expressed his satisfaction for the adequate presence of Italian mathematical activity in the international community. However, he drew the Congress' attention to the necessity of elaborating new policies and a new structure for the development of mathematics, in order to further enlarge the research fields and to think over the teaching methods, starting from the primary school stage.

His activity in UMI was directed, in particular, to the problem of renovating Italian Mathematics "after the sterile closure of dictatorship and war"[41], whose damaging effects he had experienced personally at the beginning of his scientific career. The promotion of research was very dear to Stampacchia. He followed with great interest every initiative which contributed towards this end – for example, the program of visiting professors promoted by the National Committee for Mathematics of the National Research Council, and the CIME (International Mathematical Summer Center), which favoured the insertion of Italians in the international mathematical community, with the help of a program of updating and meetings – and he urged the Ministry of Education to pass the law on the reorganization of the Istituto Nazionale di Alta Matematica, which had been put under a commissioner since 1962. He gave special attention to the journal Bollettino dell'UMI, which, during the period of his Presidency, enhanced its prestige and also enlarged its diffusion.

Being highly sensitive to the problems of the younger generation, he promoted several initiatives in order to direct young people towards mathematics and he was always concerned about the difficulties encountered by graduates in mathematics to obtain adequate jobs. He followed the questions related to the approval of the University Reform Bill by the Parliament and he proposed the institution of a commission to study the problems of teaching mathematics during the first two years of the university courses. Through the Italian Commission for the Teaching of Mathematics, he also took interest in teaching at the secondary school level. Accordingly, he favoured contacts between universities and secondary schools, as well as the diffusion in Italy of modern trends of teaching mathematics in foreign countries.

Even after being fully involved in the activities of UMI and with the University of Pisa, Stampacchia undertook many visits abroad, especially to France and to the United States. Between the years 1965 and 1968 he made numerous visits to Paris, to the University of California at Berkeley, to the University of Minnesota at Minneapolis and to the University of Chicago. The publication of joint papers with Jacques Louis Lions, Philip Hartman, Haïm Brezis and Hans Lewy goes back to the period of these visits.

Starting from this period, his research activity was concentrated on the theory of variational inequalities.[42] Stampacchia took the variational theory of boundary value problems for partial differential equations as the model: variational inequalities, in fact, represent a very natural generalization of such problems and allow one to consider several questions arising in such different contexts as Mechanics, Physics and Convex Programming.

Continuing the studies he begun in 1964, Stampacchia announced (*Ma*[49]), in collaboration with Lions, the generalization to not necessarily coercive bilinear forms of his first result on this matter published in Comptes Rendus (*Ma*[43]). Associating some of the techniques already used in the study of minimum problems (*Ma*[38]) to the first results obtained for variational inequalities, he analyzed with Hartman the existence, uniqueness and regularity of solutions of the Dirichlet problem for a nonlinear equation with a term depending functionally on the unknown (*Ma*[51]). The result was based on a preliminary theorem where conditions were given for the existence of solutions of a variational inequality related to a monotone operator.[43]

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[40] G. Stampacchia, *Discorso inaugurale*, Atti IX Congresso UMI, 1971 (1974), p. 6.
[41] G. Stampacchia, *Discorso*, Atti VIII Congresso UMI, 1967 (1968), p. 18.
[42] For a detailed presentation of the main contributions of G. Stampacchia to the theory of variational inequalities see Lions 1978.
[43] A similar result was independently obtained by Felix Browder, *Non linear monotone operators and convex sets in Banach spaces*, Bull. Amer. Math. Soc., 71, 1965, pp. 780-785.

In another important paper with Lions,[44] published in Communications on Pure and Applied Mathematics (*Ma*[53]), Stampacchia re-examined the entire linear theory, studying variational inequalities associated to bilinear forms which are coercive or simply non negative in Hilbert spaces, with applications to elliptic and parabolic operators and to problems with unilateral constraints. The regularity of the solutions of problems with obstacles for a second order linear elliptic operator, and the nature of the set of contact with the obstacle, were studied jointly with Hans Lewy (*Ma*[59]). This paper, published in 1969, was presented at the VIII UMI Congress in 1967 (*Ma*[60]). Finally, in collaboration with Brezis, Stampacchia obtained an abstract regularity theorem for variational inequalities associated to non linear monotone operators, a theorem applicable to a number of examples wherein the convex sets are defined by constraint conditions on the unknown function or on its gradient[45] (*Ma* [57]). The application of the theorem of Brezis-Stampacchia to the case of a convex set defined via an obstacle from above and an obstacle from below was considered in Stampacchia's lecture at the American Mathematical Society Conference, Nonlinear Functional Analysis, held in Chicago in April 1968 (*Ma*[62]).

On an invitation from Aldo Ghizzetti, Stampacchia gave an advanced level course on Variational Inequalities at the NATO Summer School on Theory and Applications of Monotone Operators held in Venice during the summer of 1968. The lecture notes of this course (*Ma*[58]), which are a re-elaboration of the lecture notes for a course delivered in Minneapolis during the spring of the same year, contain a clear exposition of the results on existence and regularity of solutions of variational inequalities obtained up to that time, together with many examples and applications.

4. At the University of Rome "La Sapienza": the Student Movement, the appointment as Director of the Istituto per le Applicazioni del Calcolo.

Following informal contacts started in February 1967, in November 1968 Stampacchia, the only person whose prestigious name drew complete unanimity among the professors of the then Mathematics Institute, was invited to take the position of Professor of Mathematical Analysis I (second chair) by the Faculty of Sciences of the University of Rome "La Sapienza". He was assigned the course of Istituzioni di Analisi Superiore in the academic year 1968-69, the course of Analisi Superiore in the following year.

In the spring of 1970 he attended the ceremony of dedication to Beppo Levi of the Institute of Mathematics of the University of Rosario, in Argentina; that year he also visited the University of Sussex and the following year the University of Maryland at Baltimore.

It is not difficult to imagine the influence that Stampacchia, who had brought with him a group of Italian as well as foreign young researchers, could have had in Rome both scientifically and also in the field of teaching: there was a lot of hope in this direction. But, instead, because of unfavourable circumstances and conditions, his stay in Rome lasted only two years, after which he accepted with pleasure the invitation to transfer himself to the Scuola Normale at Pisa.

During the winter of 1970 "La Sapienza" was the theater of violent contrasts between the Student Movement and the neo-fascist group of Avanguardia Nazionale, fighting each other to gain control of University politics. There were frequent assemblies, demonstrations and occupations of the buildings; often, the police was called in by academic authorities, to intervene inside the University campus in order to separate the opposing factions or to disperse the demonstrating crowds. In this atmosphere Stampacchia assumed a position against the extreme right student organizations and publicly protested against the fascist demonstrations inside the campus. In a letter sent to the Dean of the Faculty of Sciences, Prof. Montalenti, Stampacchia expressed all his

[44] See Lions 1978, pp. 740-741 (XXXVII-XXXVIII).
[45] See Lions 1978, pp. 747-748 (XLIV-XLV).

indignation for the facts he have had to watch, and announced his decision not to go to the Institute of Mathematics until every form of apologetic behaviour ceased. Among other things, he wrote: "Illustrious Dean, it is nearly a year and a half that I have been carrying out my duties as a member of the teaching staff in your Faculty. I have seen episodes of struggle by the Student Movement which I first followed in the hope that it would act as an incentive for the indispensable reforms in the university system; then with a certain amount of mistrust, when these demonstrations became simply a demand for more examinations and less lessons; finally, with a lot of discomfort when a part of the teaching staff, self-defining subordinate, joined this movement, looking for positions of power only alternative to that of the so called barons. ... Unfortunately, while entering the Città Universitaria (University campus) to go to the Institute, I have been forced, for some days, to listen to hymns and to see walls filled with symbols inspired by those harmful ideologies of which our generation very well knows the disastrous consequences. I refer to those ideologies in whose name, incapable, corrupt and violent men rose to power and removed from their teaching positions, and quite often cancelled, the best intelligentsia, lowering the cultural and social standards of Italy and exposing our country to the contempt of the whole world. If I were to continue to teach in this Città Universitaria under these conditions, I would feel responsible for betraying not only my conscience, but also the memory of all those who fought those barbaric ideologies and who worked to lift up Italy from the level to which she had fallen. Therefore I am forced to announce that, until the academic authorities rid the Città Universitaria of the groups of these hooligans, whose objectives are the denial of culture and of any social renovation, it will not be possible for me to go to the Institute of Mathematics."[46]

This decision of not to teach any more at the Institute of Mathematics was referred to in the daily newspapers of Rome as well as in national newspapers, some of which also carried ample sections of the letter to the Dean. The same day two self−styled students of Mathematics sent an insulting letter to Stampacchia, which he always kept among his papers.

It Rome Stampacchia was entrusted with the direction of the Istituto per le Applicazioni del Calcolo (IAC) of the National Research Council (CNR), in December 1968. He took office with the precise aim of strengthening and updating the research sector of the Institute, by taking on young graduates as research workers, by appointing consultants of high level scientific competence and by encouraging visits of foreign fellowship holders and expert scholars, among whom we find the distinguished names of Lars Hörmander and Hans Lewy.

With Hans Lewy he studied the question of the regularity of the solution of the obstacle problem for the Laplacian, by considering it as the minimum of the superharmonic functions which respect the constraint (*Ma*[65]) and focused his attention on nonlinear monotone operators defined by means of a vector field, proving, in the case of a Lipschitz continuous obstacle, the existence of a Lipschitz continuous solution which may exhibit further regularization properties (*Ma*[66]). This type of problems contains as a particular case the problem of a minimal surface which lies over an obstacle and assumes fixed boundary values, for which the study of the contact set was later continued by David Kinderlehrer.[47]

Variational inequalities were also the subject of his talks in two international meetings of great relevance. He was one of the invited speakers at the International Conference on Functional Analysis and Related Topics organized in Tokyo by the International Mathematical Union and the Mathematical Society of Japan, in April 1969. In his talk (*Ma*[63]) he gave an exposition of the theorems of existence and of regularity he had obtained with Hartman, Brezis and Hans Lewy, followed by a treatment of several examples. He gave a special lecture (*Ma*[63]) at the International Congress of Mathematicians held in Nice in September 1970, where he presented the main techniques used in problems of variational inequalities, such as the method of penalization for

[46] Paese Sera, March 2nd, 1970, p. 4; Corriere della Sera, March 3th, 1970, p. 7.
[47] D. Kinderlehrer, *How a minimal surface leaves an obstacle*, Acta Mathematica 130, 1973, pp. 221-242.

approximating the solution, the use of the lemma of Minty for passing to the limit, and the introduction of pseudo-monotone operators due to Brezis.

The main guiding principle that inspired him in the activities at the Istituto per le Applicazioni del Calcolo was his belief, expressed on several occasions, that the distinction between pure and applied mathematics, being very vague and variable with time, was artificial. In his own words: "To take a just attitude towards mathematics it is necessary to reject a distinction (because it is dangerous) between pure mathematicians and applied mathematicians."[48] He was also very concerned with the influence that powerful industrial groups, increasing in number, could exercise to direct the research work, thus undermining the freedom of science from economic power: "The life of research centers and of advanced teaching institutions is necessary at the present moment, in which Universities seem to be unable to carry out this task. The task of research and of preparation of qualified research workers could have been carried out by the National Research Council through some of its institutes. But this task seems now difficult to realize also because a good deal of the funds that the Country has invested for scientific and technological research will end up funding research activities more industry related, thus neglecting fundamental research which would have permitted the formation of real research workers, and not just simple users of national or foreign industrial products."[49]

The experience at the IAC came to an end in January 1971 when, during the time of a visit of three months to Berkeley, a Commissioner was nominated by the President of CNR to substitute him as the director of the Institute. Stampacchia experienced a great bitterness because of this event and made every effort to remove the shadow that he felt he was under due to his dismissal – even filing a complaint before the Consiglio di Stato (Supreme Administrative Court). This matter ended only in 1975, when Sandro Faedo having become the President of CNR, both parties reached an agreement which Stampacchia, in a letter, considered "satisfactory not so much from the material point of view but rather for my moral condition, since our own past vicissitudes influence the future of every one of us. When one reaches my age, when one thinks of having given to scientific culture as much as one could give, with the impression that in the future one will no more have similar opportunities, at this age it is a pleasure to receive also formal acknowledgements. If instead of this, one suffers an abuse of power, believe me, one feels very depressed."[50]

On reading again Stampacchia speeches, one is often impressed with the extreme lucidity and incredible up-to-dateness of some of his statements: in the introductory report prepared in September 1973 for the Congress "Relations between pure and applied mathematics in Italy", with the aim to denounce the bureaucratization of research and of scientific policies in Italy, he renamed the CNR, National Council for Research, as National Council for Reports, and defined "Comprogresso"("Comprogress"), the process by which the reforms are often carried out in Italy. He wrote: "One of the fundamental processes put in execution in Italian life can be baptised with the name Comprogress. It is the result of different demands of progress very much heard of, and of their compromise. One of the illustrious victims of Comprogress is the Italian University."

5. Once again at Pisa: back to the Scuola Normale.

Having been invited, with a unanimous vote of the Council of Direction of the Scuola Normale Superiore, including the student representatives, to assume the position of professor for the chair of Analisi Superiore, Stampacchia returned to Pisa starting on November 1st, 1970.

[48] G. Stampacchia, *Matematica pura e applicata negli sviluppi attuali*, introductory report to the Congress "Rapporti tra ricerca pura ed applicata in Italia", Siena, 27-29 September 1973.
[49] Autograph preparatory manuscript for the inaugural speech for the IX UMI Congress, Bari, 1971.
[50] G. Stampacchia to his lawyer M. Tarello, 29 June 1973, rough draft of an autograph letter.

Together with his colleagues De Giorgi and Vesentini, he strove to maintain the international prestige reached by the Scuola Normale and to create a lively scientific atmosphere with frequent visits of foreign mathematicians, among which we mention the prestigious names of J. Leray, H. Lewy, J.L. Lions, L. Nirenberg, O. Oleinik, D.G. Aronson, H. Brezis, R. Finn, J. Serrin, M.F. Atiyah, and D. Edmunds. Moreover, he found a new pleasure in teaching: in addition to his course on Analisi Superiore, he taught Analisi Matematica to the students of Physics at the University and then, he also had the charge of the course of Matematica I for the second year students at the Scuola Normale.

We have valuable notes of some of his courses at the Universities of Pisa, of Rome and at the Scuola Normale, notes which have been used partially for his books. We recall, in particular, the notes from his lectures on second order elliptic equations of 1963-64, those of 1967-68 on the theory of ordinary differential equations, which later became the subject of a joint volume with L. C. Piccinini and G. Vidossich (*Ma*[85]), and finally the notes on variational inequalities of 1970-71.

During these years his scientific activity was devoted to variational inequalities. During his stay at Berkeley in 1971, he studied with Alfonso Vignoli, a problem with non Lipschitz continuous obstacle (*Ma*[67]) and in his talk[51] at the Conference on Theory of Ordinary and Partial Differential Equations, held in Dundee, Scotland, in March 1972, he made a survey of the recent results obtained by him and his school. In Pisa, in collaboration with Murthy, he considered an obstacle problem with mixed boundary conditions for a second order elliptic operator (*Ma*[68]). In this paper, which he presented at the International Symposium on Partial Differential Equations and the Geometry of Normed Linear Spaces, held in Jerusalem in June 1972, the regularity of the solution was proved making use preliminarily of the truncation method and then of some nonlinear approximations. Later, in September 1972, in a joint paper with Brezis and Nirenberg, he obtained the extension of some existence results for variational inequalities to the case of functions of two variables defined in a Cartesian product $C \times C$ of a convex set C in a topological vector space, and a minimax principle (*Ma*[71]).

In addition to the theoretical aspects examined so far, Stampacchia had considered also numerical and applied aspects of variational inequalities, both as a consultant of the Istituto per l'Elaborazione dell'Informazione of CNR at Pisa and in some of his publications and internal reports. With Otello Mancino, he studied variational inequalities for monotone operators on convex sets in finite dimensional spaces and gave an algorithm to find the solution of the convex programming problem in a polyhedron (*Ma*[64]).[52] At the Convegno di Analisi Numerica, held in Rome at the Istituto Nazionale di Alta Matematica in January 1972, he examined the problem of the numerical treatment of variational inequalities with an obstacle, approximating the solution with the help of two sequences of solutions of nonlinear equations, one increasing and the other decreasing, thus allowing to estimate the error introduced in the approximation (*Ma*[69]).

A very important feature of variational inequalities associated to elliptic differential operators is their connection with free boundary value problems, a connection put in evidence in Stampacchia's first paper with Hans Lewy (*Ma*[59]) and expounded in his talk at the Conference “Metodi Valutativi della Fisica Matematica” held in Rome at the Accademia dei Lincei in December 1972 (*Ma*[73]). In this talk Stampacchia described the clever trick with which Claudio Baiocchi, in 1971, had transformed the problem of filtration through a porous dam into one of variational inequality.[53] In addition, he took up the study of the stationary irrotational subsonic plane motion of a compressible fluid around a symmetric convex profile, which he had reduced to a

[51] G. Stampacchia, *Recent results in the theory of variational inequalities*, Lecture Notes in Math., 280, 1972, pp. 147-153.
[52] The paper Ma [74] collects together the notes of April 1973, taken during a seminar of Stampacchia on this work.
[53] C. Baiocchi, *Su un problema di frontiera libera connesso a questioni di idraulica*, Ann. Mat. Pura Appl., 92, 1972, pp. 107-127. A preliminary note was published in 1971 in Comptes Rendus of the Academie des Sciences of Paris.

free boundary problem associated to a variational inequality in the hodograph plane,[54] in a joint paper with Brezis (*Ma*[72]). This problem had been proposed by the Department of Rational Mechanics of the Politecnico di Torino to the Istituto per le Applicazioni del Calcolo; the interest of the solution method lies in the fact that it allows also the numerical calculation of the solution. Also the Exposé at the Séminaire Goulaouic-Schwartz at the École Polytechnique, in December 1972, was dedicated to questions of this type studied in a joint paper with Brezis. This exposition takes up the method of Baiocchi and investigates the motion of a fluid around a convex profile in the incompressible case. This latter study, appropriately developed and completed, was presented at the Conference Sur les Applications de l'Analyse Fonctionnelle aux Problèmes de Mécanique, held in Marseille in September 1975[55], and eventually published in the Archive for Rational Mechanics and Analysis in 1976 (*Ma*[78]).

During these years Stampacchia drew up the project to write a book on variational inequalities, later carried out in collaboration with D. Kinderlehrer (*Ma*[86]).

After leaving the presidency of UMI, Stampacchia occupied himself actively with the task of promoting mathematical research. He accepted the nomination as Director of the Scuola Superiore di Analisi matematica of the International Center of Scientific Culture Ettore Maiorana at Erice. On taking this office, he indicated as his main objective that of encouraging contacts among Italian and foreign specialists in various disciplines and between specialists and young research workers. To this purpose, he was one of the organizers of two courses on the theory, development and recent applications of variational inequalities. The first was held at Erice in March 1975, the second, in 1978, unfortunately turned out to be the first Conference dedicated to his memory. Moreover, he took part as teaching member, in the course on Mathematical and Numerical Methods in Fluid Dynamics, organized by the International Center for Theoretical Physics at Trieste.

There was a forced period of rest in the autumn of 1973, due to serious heart problems; but within a few months he returned to his intense teaching and scientific activities, and also to frequent visits abroad, ignoring the advice of his doctors to entertain a more relaxed and restful life style.

In November 1973 Stampacchia received, with great personal satisfaction, the invitation to give a talk on Hilbert's twenty-third problem, extensions of the Calculus of Variations, at the Mathematical Symposium on Developments Arising from Hilbert's Problems promoted by the American Mathematical Society at De Kalb, Illinois. In his lecture given on 16 May 1974, he described the history of Calculus of Variations starting from the problem of solids with minimum resistance – considered by Newton at the end of the 17th century – to the Dirichlet principle, from the contributions of Beppo Levi, Fubini and Lebesgue in Rendiconti del Circolo Matematico di Palermo to the development of direct methods. Further he examined the relation with elliptic equations and the extension of variational methods, from the theory of partial differential equations to variational inequalities, from non linear functional analysis to problems of optimal control (*Ma*[79]).

He took the Editorial Directorship of the Science Section of the Annali della Scuola Normale Superiore in 1974. His first task was to include in the editorial committee mathematicians of great reputation from different countries such as S. Agmon, J. Leray, J.L. Lions, L. Nirenberg, in order to raise the journal to high international standards and prestige. At that time he was also a member of the editorial committees of Advances in Mathematics, Applied Mathematics and Optimization, and of Calcolo, and he was a member of the selection committee for invited speakers, presided over by Jean Leray, at the International Congress of Mathematicians held at Moscow (1966) and then at Vancouver (1974).

[54] See Lions 1978, pp. 794-750 (XLVI-XLVII).

[55] H. Brezis - G. Stampacchia, *The odograph method in fluid-dynamics in the light of variational inequalities*, in: Applications of methods of functional analysis to problems in mechanics, Lecture Notes in Math., 503, 1976, pp. 239-257.

Stampacchia kept his interest in free boundary problems and considered, in a paper dedicated to the memory of Ivan G. Petrovskii, the motion of a fluid in a porous medium, for instance, water in an earth dam, in a model which could not be reduced to a two dimensional problem (*Ma*[75]). He reduced the three-dimensional problem to a variational inequality, and established the regularity of the solution; furthermore, he showed that the free surface is the graph of a function, leaving open the problem of its regularity, which was later on treated in a general way by Hans W. Alt.[56] In addition to his studies on the regularity of solutions of variational inequalities associated to second order operators recalled so far, Stampacchia considered also operators of fourth order, and established the regularity of solutions of some variational inequalities.[57] As a first step, he examined a one-dimensional model representing the elastoplastic behaviour of a beam, in a paper dedicated to G. Sansone for his eightyfifth birthday (*Ma*[76]); then he studied, in a joint paper with Brezis, a variational inequality for the biharmonic operator in *n* variables (*Ma*[80]).

In August 1975 he was invited to the Netherlands for the second Scheveningen Conference on New Developments in Differential Equations. There he presented a joint work with Kinderlehrer[58] where they examined a free boundary problem for the Poisson equation in the plane through a variational inequality approach and they proved that the unknown curve, on which conditions of both Dirichlet and Newman types are simultaneously imposed, is regular (*Ma*[77]). Finally, a joint paper with Brezis and Kinderlehrer, which was published posthumously, was once again dedicated to the problem of filtration through a porous dam.[59] In this article, a new formulation was proposed for the problem, whose solution coincides with that of Baiocchi, in the case of a rectangular dam (*Ma*[81]).

Stampacchia also wrote textbooks for university courses and articles for popular scientific publications, such as the ones issued by Istituto Geografico De Agostini and Enciclopedia Einaudi.

His total rejection of any superficially demagogical attitude, his feeling of distrust and his pessimism for the future of the Italian university system, which had already characterized his choices and his speeches, were once more evident towards the end of 1977. At that time, in a renewed climate of violence and uncertainty, he was called upon to preside over a Ministerial Investigating Commission, to evaluate the behaviour of a teacher of Mathematical Analysis and Analytical Geometry at the Faculty of Architecture of the Polytechnic of Milano, where several irregularities had been denounced. The commission, in the report presented to the Minister, interpreted the facts that had taken place as “a symptom of the great uneasiness that one perceives in our universities” after more than ten years of waiting for an organic reform law, and remarked that the legislative measures which had been passed had created “a situation far worse than that which was intended to be remedied”. In a handwritten draft, Stampacchia judged the attitude of the entire faculty as “an affront to those who want to defend the principle of the autonomy of the university system, a principle on which the University, its progress and its cultural tradition are based."

He spent a considerable amount of his energies also to make known abroad his research activity, giving seminar talks on the theory of partial differential equations and variational inequalities: he was a visiting professor at the University of Sussex from October to December of 1971 and in the month of February 1976 and he spent a month in Paris at Collège de France between May and June 1976. He also went to the United States for three months from March to May of 1977 as visiting professor at the Courant Institute in New York and at the School of Mathematics of the University of Minnesota at Minneapolis. At last he returned to Paris around the

[56] H.W. Alt, *The fluid flow through porous media. Regularity of the free surface*, Manuscripta math., 21, 1977, pp. 255-272.
[57] See Lions 1978, pp. 750-751 (XLVII-XLVIII).
[58] G. Stampacchia, *Free boundary problem for Poisson’s equation*, New developments in differential equations, 1976, pp. 39-42.
[59] See Lions 1978, pp. 751-752 (XLVIII-XLIX).

middle of February 1978, as a visiting professor for two months at the University Pierre et Marie Curie, to give a course on partial differential equations.

In Paris he suffered once more a serious heart attack, following which he was admitted to Boucicaut Hospital. Just when the evolution of the illness seemed to be taking such a satisfactory course that he was authorized to return home, Stampacchia expired due to a sudden heart arrest, on 27 April 1978, the same day he was to be discharged from the hospital. According to a wish he had expressed many times, he was buried in the British Cemetery in Naples.

On 17 May 1978, the Council of Directors of National and International Schools of the Centro di Cultura Scientifica Ettore Maiorana at Erice decided to honour his memory by dedicating to his name the International School of Mathematics, which hosts the present Conference, and by founding a fellowship to be awarded to a young mathematician to attend the School.

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