

In the past few years there has been an enormous change in the way that wholesale prices for electricity are determined in many parts of the world. Increasingly market mechanisms are being set up in which the clearing price for electricity is determined by some sort of auction process. The book by Chao and Huntington [6, 12] gives a useful overview of the nature of electricity markets and the working paper by von der Fehr and Harbord [7] is another useful starting point as well as reviewing the form of a variety of markets as they existed in 1998.

Recent problems in the Californian electricity market have had enormous costs, and have been attributed to poorly designed market mechanisms by some commentators.

A typical electricity wholesale market operates as follows: generators are requested to offer in price quantity stacks in an ascending order to a central dispatching authority usually one day ahead. Market demand realizes and the market clearing price is determined. All offers below the market clearing price are dispatched and paid at the clearing price. This model has to be adjusted when demand and supply at different points in a network. In this case there will be different clearing prices at different parts of the network. Once the offers are made and the demand is known, the central dispatching authority solves a linear program to determine the actual dispatches from each generator.

Generators have complex problems to solve when they make offers into the markets. There are uncertainties in market demand and in other generators' behaviour. Optimization and game theory have been useful in modeling a generator's bidding strategy and interactions among generators.

The notion of supply function equilibria developed by Klemperer and Meyer [9] for a general oligopoly market and applied by Green and Newbery [8] to British electricity markets is a successful application of game theory in electricity market. In a supply function equilibria model, each generator offers into a wholesale spot market a smooth increasing supply function which is optimal in every realization of market demand. One of the interesting questions is the existence of Nash equilibria at which no generator can increase his profit by unilaterally changing his offer. A Nash equilibrium is usually characterized by a single differential equation or a set of differential equations. There have been many papers addressing this issue following Green and Newbery's work; see [10, 11, 4, 5] and the references therein.

It is sometimes argued that the smooth supply function model may over

simplify a generator's offer. For instance, in Australian and New Zealand markets, generators are restricted to offer a set of quantities at distinct prices (an "offer stack"). Anderson and Philpott [1] consider a model where generators' supply functions may be step-like. They model a generator's expected profit as a line integral of the generator's profit function with respect to a probability function along the generator's supply curve. The resultant problem becomes an infinite mathematical programming problem. This work has been extended by Anderson and Xu [2, 3].

References

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